Lagrange's Interpolation Formula

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Lagrange's Interpolation Formula is determined as a special case of polynomial transposition [2].

A number of points is determined with unique locations x_j.

$$y_j = f(x_j); \qquad 0 \le j < n \tag{1}$$

Therefore an interpolation polynomial is determined by as many terms.

$$y = f(x) = \sum^{0 \le i < n} a_i * x^i \tag{2}$$

Every point is assigned a base polynomial or weight w_j. Suppose the sum of all weighted conditions equals the polynomial.

$$f(x) = \sum^{0 \le i < n} a_i * x^i = \sum^{0 \le j < n} w_j * y_j = \sum^{0 \le j < n} w_j * \sum^{0 \le i < n} a_i * x_j^i \tag{3}$$

The double sum is interchanged.

$$f(x) = \sum^{0 \le i < n} a_i * x^i = \sum^{0 \le j < n} w_j * y_j = \sum^{0 \le i < n} a_i * \sum^{0 \le j < n} w_j * x_j^i \tag{4}$$

The base polynomials are determined by a system of linear equations according to a comparison by coefficients.

$$\sum^{0 \le j < n} w_j * x_j^i = x^i; \qquad 0 \le i < n \tag{5}$$

The base matrix is a transposed Vandermonde matrix.

$$G = \sum^{0 \le i < n} \left\langle \sum^{0 \le j < n} \left\langle x_j^i \right\rangle \right\rangle \tag{6}$$

The determinant of a Vandermonde matrix equals the product of all possible differences. The determinant is non-zero if all locations are unique.

$$\det(G) = \prod^{1 \le i < n} \prod^{0 \le j < i} (x_i - x_j) \tag{7}$$

A base polynomial is determined by Cramer's rule. Thus a source matrix is a variant of the base matrix for which one column is replaced by the source. The determinant of a source matrix is determined accordingly.

$$\det(Q_m) = \prod^{1\leq i<n} \prod^{0\leq j<i} \begin{cases} x - x_j, & \text{if } i = m \\ x_i - x, & \text{if } j = m \\ x_i - x_j, & \text{otherwise} \end{cases} \tag{8}$$

A base polynomial is determined by Cramer's rule. A number of differences and signs cancel.

$$w_j = \frac{\det(Q_j)}{\det(G)} = \frac{\prod\limits_{i\neq j}^{0\leq i<n} (x_i - x)}{\prod\limits_{i\neq j}^{0\leq i<n} (x_i - x_j)} \tag{9}$$

Lagrange's Interpolation formula is determined by polynomial transposition.

$$f(x) = \sum^{0\leq j<n} w_j * y_j \tag{10}$$

References

[1] Lagrange's Interpolation Formula, Hans-Dieter Reuter, http://www.joinedpolynomials.org/lagrange.pdf

[2] Joined Polynomials, Hans-Dieter Reuter, http://www.joinedpolynomials.org/jp.pdf